Financial Economic Theory and Engineering Formula Sheet

2011

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee felt that by providing many key formulas, candidates would be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula sheet was developed sequentially by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. For example, there are several versions of the Black-Scholes-Merton option pricing formula to differentiate between instruments paying dividends, tied to an index, etc. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

General Formulas

Lognormal distribution:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(\ln(x)-\mu)^2}{\sigma^2}\right\}$$

$$\mathrm{E}[X] = e^{\mu+\sigma^2/2} \qquad \mathrm{V}[X] = e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$$

$$\Pr[X \le x] = \Phi\left(\frac{\ln(x)-\mu}{\sigma}\right) \quad x > 0$$

Normal distribution:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$$

Capital Asset Pricing Model:

$$\mathrm{E}[R_i] = R_f + \beta_i\,(\mathrm{E}[R_m] - R_f) \qquad \text{where} \quad \beta_i = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}[R_m]}$$

Weighted Average Cost of Capital:

$$\mathrm{WACC} = (1-\tau_c)k_b\frac{B}{B+S} + k_s\frac{S}{B+S}$$

where k_b=return on debt, k_s =return in equity.

GARCH(1,1) Model

$$Y_t = \mu + \sigma_t\epsilon_t \qquad \sigma_t^2 = \alpha_0 + \alpha_1(Y_{t-1}-\mu)^2 + \beta\sigma_{t-1}^2$$

RSLN-2 Model

$Y_t|\rho_t \sim N\left(\mu_{\rho_t}, \sigma_{\rho_t}^2\right)$ where $\{\rho_t\}$, $t = 1, 2, \ldots,$ is a Markov process with 2-states.

$$p_{ij} = \Pr[\rho_{t+1} = j|\rho_t = i]$$

Conditional Tail Expectation

For loss L, continuous at $V_\alpha = F_L^{-1}(\alpha)$:

$$\mathrm{CTE}_\alpha(L) = \mathrm{E}\left[L|L > V_\alpha(L)\right]$$

If there is a probability mass at V_α, define $\beta' = \max\{\beta : V_\alpha = V_\beta\}$, then

$$\mathrm{CTE}_\alpha(L) = \frac{(1-\beta')E\left[X|X > V_\alpha\right] + (\beta'-\alpha)V_\alpha}{1-\alpha}$$

Vasicek Model

$$\begin{aligned} dr &= a(b-r)dt + \sigma dz \qquad P(t,T) = A(t,T)e^{-B(t,T)r(t)} \\ B(t,T) &= \frac{1-e^{-a(T-t)}}{a} \qquad A(t,T) = \exp\left[\frac{(B(t,T)-T+t)\left(a^2b-\sigma^2/2\right)}{a^2} - \frac{\sigma^2 B(t,T)^2}{4a}\right] \end{aligned}$$

Cox Ingersoll Ross Model

$$\begin{aligned} dr &= a(b-r)dt + \sigma\sqrt{r}dz \qquad P(t,T) = A(t,T)e^{-B(t,T)r(t)} \\ B(t,T) &= \frac{2(e^{\gamma(T-t)}-1)}{(\gamma+a)(e^{\gamma(T-t)}-1)+2\gamma} \qquad A(t,T) = \left[\frac{2\gamma e^{(a+\gamma)(T-t)/2}}{(\gamma+a)(e^{\gamma(T-t)}-1)+2\gamma}\right]^{2ab/\sigma^2} \end{aligned}$$

Ho-Lee Model

$$\begin{aligned} dr &= \theta(t)dt + \sigma dz \qquad \theta(t) = F_t(0,t) + \sigma^2 t \\ P(t,T) &= A(t,T)e^{-r(t)(T-t)} \\ \ln A(t,T) &= \ln\frac{P(0,T)}{P(0,t)} + (T-t)F(0,t) - \frac{1}{2}\sigma^2 t(T-t)^2 \end{aligned}$$

Hull-White (Extended Vasicek) Model

$$\begin{aligned} dr &= (\theta(t)-ar)dt + \sigma dz \qquad \theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}\left(1-e^{-2at}\right) \\ P(t,T) &= A(t,T)e^{-B(t,T)r(t)} \\ B(t,T) &= \frac{1-e^{-a(T-t)}}{a} \\ \ln A(t,T) &= \ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4a^3}\sigma^2\left(e^{-aT}-e^{-at}\right)^2\left(e^{2at}-1\right) \end{aligned}$$

Itô's Lemma

Let X be an Itô process such that $dX_t = u(t,X_t)\,dt + v(t,X_t)\,dW_t$,

and let $g(t,x)$ denote a twice differentiable function, then for $Y_t = g(t,X_t)$

$$dY_t = \frac{\partial g(t,X_t)}{\partial t}\,dt + \frac{\partial g(t,X_t)}{\partial x}\,dX_t + \frac{1}{2}\frac{\partial^2 g(t,X_t)}{\partial x^2}\,(dX_t)^2.$$

Geometric Brownian Motion

$$\frac{dS_t}{S_t} = \mu\,dt + \sigma\,dZ_t$$

Black-Scholes Pricing Formulas

$$c_t = S_t N(d_1) - Ke^{-r(T-t)}N(d_2)$$

$$p_t = Ke^{-r(T-t)}N(-d_2) - S_tN(-d_1)$$

where $d_1 = \dfrac{\ln(S_t/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$ and $d_2 = d_1 - \sigma\sqrt{T-t}$

Copeland, Weston, Shastri: Financial Theory and Corporate Policy

Chapter 2

(2.2) $Rev_t + m_t S_t = Div_t + (W\&S)_t + I_t$

(2.5) $NI_t = Rev_t - (W\&S)_t - dep_t$

(2.7) $S_0 = \sum_{t=1}^{\infty} \frac{NI_t - \Delta A_t}{(1+k_s)^t}$

(2.13) $FCF = EBIT(1-\tau_c) + \Delta dep - \Delta I$

Chapter 6

(6.34) $R_{jt} = E(R_{jt}) + \beta_j \delta_{mt} + \varepsilon_{jt}$

(6.36) $R_{jt} - R_{ft} = (R_{mt} - R_{ft})\beta_j + \varepsilon_{jt}$

(6.36) $R'_{pt} = \gamma_0 + \gamma_1 \beta_p + \varepsilon_{pt}$

(6.37) $R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$

(6.38) $E(R_i) = E(R_Z) + [E(R_m) - E(R_Z)]\, \beta_i$

(6.40) $E(R_i) - R_f = b_i\, [E(R_m) - R_f] + s_i E(\text{SMB}) + h_i E(\text{HML})$

(6.41) $\lambda_i = E(R_m) - E(R_Z)$

(6.46) $R_{it} = \widehat{\gamma}_{0t} + \widehat{\gamma}_{1t} \beta_{it} + \varepsilon_{it}$

(6.49) $E(R_i) = E(R_{z,I}) + [E(R_I) - E(R_{z,I})]\, \beta_{i,I}$

(6.50) $\widetilde{R}_i = E(\widetilde{R}_i) + b_{i1}\widetilde{F}_1 + \ldots + b_{ik}\widetilde{F}_k + \widetilde{\varepsilon}_i$

(6.57) $E(\widetilde{R}_i) = \lambda_0 + \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik}$

(6.59) $E(R_i) - R_f = \left[\bar{\delta}_1 - R_f\right] b_{i1} + \ldots + \left[\bar{\delta}_k - R_f\right] b_{ik}$

(6.60) $b_{ik} = Cov(R_i, \delta_k)/Var(\delta_k)$

Chapter 9

(9.1) $C(S_A, S_B, T) = S_A N(d_1) - S_B N(d_2)$

where $d_1 = \left[\ln(S_A/S_B) + V^2 T\right]/V\sqrt{T}$; $d_2 = d_1 - V\sqrt{T}$
and $V^2 = V_A^2 - 2\rho_{AB} V_A V_B + V_B^2$

Chapter 10

(10.1) $V(\eta) \equiv \sum_m q(m) \max_a \sum_e p(e|m) U(a,e) - V(\eta_0)$

(10.4) $$p(r-c_2)+(1-p)(dr-c_2)=p\,(r/d-c_1)+(1-p)(r-c_1)$$

(10.7) $$\text{Fair Game: } \varepsilon_{j,t+1}=\frac{P_{j,t+1}-P_{jt}}{P_{jt}}-\frac{E(P_{j,t+1}|\eta_t)-P_{jt}}{P_{jt}}=0$$

(10.9a) $$\text{Submartingale: } \frac{E(P_{j,t+1}|\eta_t)-P_{jt}}{P_{jt}}=E(r_{j,t+1}|\eta_t)>0$$

(10.9b) $$\text{Martingale: } \frac{E(P_{j,t+1}|\eta_t)-P_{jt}}{P_{jt}}=E(r_{j,t+1}|\eta_t)=0$$

(10.14) $$E(R_{jt}|\widehat{\beta}_{jt})=R_{jt}+\left[E(R_{mt}|\widehat{\beta}_{mt})-R_{ft}\right]\widehat{\beta}_{jt}$$

Chapter 11

(11.2) $$R_{jt}=\alpha_j+\beta_{1j}(R_{mt}-R_{ft})+\beta_{2j}(RLE_t-RSE_t)+\beta_{3j}(HBTM_t-LBTM_t)+\varepsilon_{jt}$$

(11.3) $$\Delta NI_{jt}=\hat{a}+\hat{b}_j\Delta m_t+\varepsilon_{jt}$$

(11.4) $$\Delta\widehat{NI}_{j,t+1}=\hat{a}+\hat{b}_j\Delta m_{t+1}$$

Chapter 12

(12.1) $$V(\alpha)=\frac{1}{(1+r)}[\mu(\alpha)-\lambda]$$

(12.3) $$W_0=X+\beta V_M+Y-(1-\alpha)V(\alpha)$$

(12.4) $$\widetilde{W}_1=\alpha[\mu+\widetilde{\varepsilon}-\mu(\alpha)+\lambda]+\beta[\widetilde{M}-(1+r)V_M]+(1+r)(W_0-X)+\mu(\alpha)-\lambda$$

(12.5) $$\frac{\partial E\left(U\left(\widetilde{W}_1\right)\right)}{\partial\alpha}=E\left[U'\left(\widetilde{W}_1\right)(\mu+\widetilde{\varepsilon}-\mu(\alpha)+\lambda+(1-\alpha)\mu_\alpha)\right]=0$$

(12.6) $$\frac{\partial E\left(U\left(\widetilde{W}_1\right)\right)}{\partial\beta}=E\left[U'\left(\widetilde{W}_1\right)\left(\widetilde{M}-(1+r)V_M\right)\right]=0 \qquad \text{where} \qquad \mu_\alpha=\frac{\partial\mu}{\partial\alpha}$$

(12.7) $$(1-\alpha)\mu_\alpha=-\frac{E\left[U'\left(\widetilde{W}_1\right)(\widetilde{\varepsilon}+\lambda)\right]}{E\left[U'\left(\widetilde{W}_1\right)\right]}$$

(12.15) $$E(D)=\frac{1}{1+r}\left[V(D)+\mu-\tau_pD-\beta\int_{\underline{X}}^{D}(X-D)f(X)dX\right]$$

(12.16) $$E(D)=\frac{1}{1+r}\left[V(D)+\frac{t}{2}-\tau_pD-\beta\frac{D^2}{2t}\right]$$

(12.17) $$V'(D^*)=\tau_p+\beta\frac{D^*}{t}$$

(12.18) $$V\left[D^*(t)\right]=\frac{1}{r}\left[\frac{t}{2}-\tau_pD^*(t)-\beta\frac{[D^*(t)]^2}{2t}\right]$$

(12.20) $$V[D^*(t)] = (\tau_p + \beta A)D^*(t)$$

(12.21) $$A = -\left[\frac{\tau_p}{\beta}\right]\left[\frac{1+r}{1+2r}\right] + \left[\frac{\tau_p}{\beta}\right]\left[\frac{1+r}{1+2r}\right]\sqrt{1+\frac{\beta(1+2r)}{\tau_p^2(1+r)^2}}$$

(12.22) $$I + D = C + Np_e = C + P_e$$

(12.24) $$\max_D \left(L - \tau_p D + \left[\frac{P + \tau_p D - L}{P + \tau_p D + I - C}\right] X\right)$$

(12.25) $$\tau_p = \left(\tau_p + \frac{\partial P}{\partial D}\right)\frac{L + I - C}{P + \tau_p D + I - C} X$$

(12.26) $$D(X) = \frac{1}{\tau_p}\max(I - C + L, 0)\ln X$$

(12.30) $$V_0^{old} = \frac{P'}{P' + E}(E + S + a + b)$$

(12.36) $$W^0(z) = \alpha\overline{P}(z) + \beta P - T(m, P)$$

(12.37) $$W^s(n, z) = \alpha\widehat{P}(n, z) + \beta P - T(n, p)$$

(12.38) $$W_n = \alpha\widehat{P}_n - \frac{t_2}{p^{\gamma-1}} = 0$$

(12.40) $$\widehat{P}(n, z) = k[n + c(z)]^{1/\gamma} \quad \text{where} \qquad k = (t_2\gamma/\alpha)^{1/\gamma}$$

(12.41) $$M(n, z) = \widehat{P}(n, z) - T(n, P) = k(1 - t_1)[n + c(z)]^{1/\gamma} - t_2 n k^{1-\gamma}[n + c(z)]^{(1-\gamma)/\gamma}$$

(12.45) $$V_0 = (1 - t)X_0$$

(12.46) $$E(X|n) = \widehat{X}(n) = \frac{X_0 s_0 + \widehat{Y_m}(n) s_m}{s_0 + s_m}$$

(12.47) $$V_1(n) = \widehat{X}(n) - T(n) - C$$

(12.50) $$V_2(T, \overline{Y}) = \frac{X_0 s_0 + \widehat{Y}_m s_m + \overline{Y} F T s}{s_0 + s_m + FTs} - E\left[X t(X/n) | T, \widehat{Y}_m, \overline{Y}\right] - C$$

(12.51) $$E[V_2(T)|Y_m] = \frac{X_0 s_0 + \widehat{Y}_m s_m + \left(\frac{X_0 s_0 + Y_m s_m}{s_0 + s_m}\right) FTs}{s_0 + s_m + FTs} - T - C$$

(12.54) $$\left(\frac{d\alpha}{dD}\right)[V(n) - B(n, D)] - \alpha(D)\left(\frac{\partial B}{\partial D}\right) = 0$$

(12.55) $$\left(\frac{d^2\alpha}{dD^2}\right)[V(n) - B(n, D)] - 2\left(\frac{d\alpha}{dD}\right)\left(\frac{\partial B}{\partial D}\right) - \alpha(D)\left(\frac{\partial^2 B}{\partial D^2}\right) < 0$$

(12.56) $$\alpha\left(D^*(n)\right)\left[V(n) - B\left(n, D^*(n)\right)\right] - [V(n) - I] = 0$$

(12.57) $$\left[\frac{d^2\alpha}{dD^2}(V - B) - 2\frac{d\alpha}{dD}\frac{\partial B}{\partial D} - \alpha\frac{\partial^2 B}{\partial D^2}\right]\frac{dD}{dn} = \frac{d\alpha}{dD}\left(\frac{\partial B}{\partial n} - \frac{dV}{dn}\right) + \alpha\left(\frac{\partial^2 B}{\partial n \partial D}\right)$$

(12.58) $$\widehat{\varepsilon}(\alpha, D) = \left[(a - D_0)^2 + D^2\frac{1 - \alpha}{\alpha^2}\right]$$

(12.59) $$\overline{D} = X_1 \left[\frac{\overline{\alpha}(\epsilon - \gamma)}{(1-\overline{\alpha})(1-\epsilon)\gamma} \right]$$

(12.60) $$\max_{c(s,p),a} \int \int U\left[s - c(s,p)\right] f(s,p|a) ds\, dp$$

(12.61) $$\int \int V\left[c(s,p)\right] f(s,p|a) ds\, dp - G(a) \geq \underline{V}$$

(12.63) $$\frac{U'[s - c(s,p)]}{V'[c(s,p)]} = \lambda$$

(12.65) $$\max_a U(k) + \lambda \left[\int \int V[c(s,p)] f(s,p|a) ds\, dp - G(a) - \underline{V} \right]$$

(12.68) $$\max_{c(s),a} \int U[s - c(s)] f(s|a)\, ds + \lambda \left[\int V\left[c(s)\right] f(s|a)\, ds - G(a) - \underline{V} \right]$$

$$+\mu \left[\int V\left[c(s)\right] f_a(s|a) ds - G'(a) \right]$$

(12.70) $$\frac{U'[s - c(s)]}{V'[c(s)]} = \lambda + \mu \frac{f_a(s|a)}{f(s|a)}$$

(12.73) $$\max_{c(s,p),a} \int \int U\left[s - c(s,p)\right] f(s,p|a) ds\, dp + \lambda \left[\int \int V[c(s,p)] f(s,p|a)\, ds\, dp - G(a) - \underline{V} \right]$$

$$+\mu \left[\int \int V[c(s,p)] f_a(s,p|a)\, ds\, dp - G'(a) \right]$$

(12.74) $$\frac{U'\left[s - c(s,p)\right]}{V'[c(s,p)]} = \lambda + \mu \frac{f_a(s,p|a)}{f(s,p|a)}$$

(12.75) $$\frac{U'\left[s - c(\mathbf{p})\right]}{V'[c(\mathbf{p})]} = \lambda + \mu_1 \frac{f_{a_1}(\mathbf{p}|a)}{f(\mathbf{p}|a)} + \mu_2 \frac{f_{a_1}(\mathbf{p}|a)}{f(\mathbf{p}|a)} + \ldots + \mu_n \frac{f_{a_1}(\mathbf{p}|a)}{f(\mathbf{p}|a)}$$

(12.81) $$\max_{c(s,\mathbf{p},m),a(m),m(m)} E_{s,\mathbf{p},m}\left[U\left[s - c(s,\mathbf{p},m)\right] | a(m)\right]$$

Subject to (for all m) $E_{s,\mathbf{p}|m}\left[\left[V[c(s,\mathbf{p},m)] - G[a(m)]\right] | a(m)\right] \geq \underline{V}$

(12.82) $$\frac{\partial V(X^*)}{\partial X} = \frac{\partial P(X^*)}{\partial X} - \frac{\partial C(X^*)}{\partial X} = 0$$

(12.83) $$S = \frac{(1-\beta)E(s) - \alpha - \lambda(1-\beta)\text{Cov}(s, R_M)}{1 + r_f}$$

(12.86) $$\max_{\alpha,\beta} \frac{(1-\beta)E(s) - \alpha - \lambda(1-\beta)\text{Cov}(s, R_M)}{1 + r_f}$$

$$+\mu \left(a\left[E(W) + \alpha + \beta E(s)\right] - b\left[\text{Var}(W) + 2\beta \text{Cov}(W, s) + \beta^2 \text{Var}(s)\right] - \underline{V} \right)$$

(12.90) $$\beta = \frac{\lambda a \text{Cov}(s, R_M)}{2b \text{Var}(s)} - \frac{\text{Cov}(W, s)}{\text{Var}(s)}$$

(12.94) $$B_{new} - B = D\left\{ -\left[P\left(\frac{V + dB}{D + dD}, 1, T, r_f, \sigma_V \right) \right] + P\left(\frac{V}{D}, 1, T, r_f, \sigma_V \right) \right\} = D(-P_X + P_Y)$$

Chapter 15

(15.2) $$V_U = \frac{E(\widetilde{FCF})}{\rho} \quad \text{or} \quad V_U = \frac{E(\widetilde{EBIT})(1-\tau_c)}{\rho}$$

(15.3) $$\widetilde{NI} + k_d D = \left(\widetilde{Rev} - \widetilde{VC} - FCC - dep\right)(1-\tau_c) + k_d D \tau_c$$

(15.4) $$V^L = \frac{E(\widetilde{EBIT})(1-\tau_c)}{\rho} + \frac{k_d D \tau_c}{k_b}$$

(15.6) $$V_L = V_U + \tau_c B$$

(15.9) $$\frac{\Delta V_L}{\Delta I} = \frac{\Delta S^0}{\Delta I} + \frac{\Delta S^n + \Delta B^0}{\Delta I} = \frac{\Delta S^0}{\Delta I} + 1$$

(15.11) $$\frac{(1-\tau_c)\Delta E(EBIT)}{\Delta I} > \rho\left(1 - \tau_c \frac{\Delta B}{\Delta I}\right)$$

(15.18) $$k_s = \Delta NI/\Delta S = \rho + (1-\tau_c)(\rho - k_b)\frac{\Delta B}{\Delta S}$$

(15.20) $$G = V_L - V_U = \tau_c B$$

(15.21) $$V_U = \frac{E(EBIT)(1-\tau_c)(1-\tau_{ps})}{\rho}$$

(15.23) $$V_L = V_U + \left[1 - \frac{(1-\tau_c)(1-\tau_{ps})}{(1-\tau_{pB})}\right] B \qquad \text{where } B = k_d D(1-\tau_{pB})/k_b$$

(15.28) $$\beta_L = \left[1 + (1-\tau_c)\frac{B}{S}\right]\beta_U$$

(15.33) $$R_f S^L + \lambda^*(1-\tau_c)\text{Cov}(EBIT, R_m) - \lambda^*(1-\tau_c)B\left[\text{Cov}(R_{bj}, R_m)\right]$$
$$= \text{E}(EBIT)(1-\tau_c) - E(R_{bj})B(1-\tau_c)$$

(15.38) $$dS = \frac{\partial S}{\partial V}dV + \frac{\partial S}{\partial t}dt + \frac{1}{2}\frac{\partial^2 S}{2\partial V^2}\sigma^2 V^2 dt$$

(15.40) $$r_s = \frac{\partial S}{\partial V}\frac{V}{S}r_V$$

(15.43) $$S = VN(d_1) - e^{-r_f T} DN(d_2)$$

(15.46) $$\beta_S = \frac{1}{1 - (D/V)e^{-r_f T}\left[N(d_2)/N(d_1)\right]}\beta_V$$

(15.47/8) $$k_s = R_f + (R_m - R_f)N(d_1)\frac{V}{S}\beta_V = R_f + N(d_1)(R_v - R_f)\frac{V}{S}$$

(15.52) $$k_b = R_f + (\rho - R_f)N(-d_1)\frac{V}{B}$$

Note: This is a correction to the formula in the reading.

(15.55) $$\frac{dV}{V} = \mu(V,t)dt + \sigma dW$$

(15.57) $$\frac{1}{2}\sigma^2 V^2 F_{VV}(V) + rVF_V(V) - rF(V) + C = 0$$

(15.58) $$F(V) = A_0 + A_1 V + A_2 V^{-(2r/\sigma^2)}$$

(15.59) $$V = V_B \Rightarrow B(V) = (1-\alpha)V_B \qquad V \to \infty \Rightarrow B(V) \to C/r$$

(15.61) $$B(V) = (1-p_B)C/r + p_B[(1-\alpha)V_B] \text{ where } \qquad p_B = (V/V_B)^{-2r/\sigma^2}$$

(15.62) $$V = V_B \Rightarrow DC(V) = \alpha V_B \qquad V \to \infty \Rightarrow DC(V) \to 0$$

(15.63) $$DC(V) = \alpha V_B \, (V/V_B)^{-2r/\sigma^2}$$

(15.67) $$V_L(V) = V_U(V) + T_c B(V) - DC(V) = V_U(V) + T_c B - p_B T_c B - \alpha V_B p_B$$

(15.68) $$M = (1+r)\gamma_0 V_0 + \gamma_1 \begin{cases} V_1 & \text{if } V_1 \geq D \\ V_1 - C & \text{if } V_1 < D \end{cases}$$

(15.69) $$M_a = \begin{cases} \gamma_0(1+r)\frac{V_{1a}}{1+r} + \gamma_1 V_{1a} & \text{if } D^* < D \leq V_{1a} \\ \gamma_0(1+r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1a} & \text{if } D < D^* \end{cases}$$

(15.70) $$M_b = \begin{cases} \gamma_0(1+r)\frac{V_{1a}}{1+r} + \gamma_1(V_{1b} - C) & \text{if } D^* \leq D \leq V_{1a} \\ \gamma_0(1+r)\frac{V_{1b}}{1+r} + \gamma_1 V_{1b} & \text{if } D < D^* \end{cases}$$

Chapter 16

(16.2) $$V_i(t) = \frac{Div_i(t+1) + n_i(t)P_i(t+1)}{1 + k_u(t+1)}$$

(16.8) $$\widetilde{V}_i(t) = \frac{\widetilde{EBIT}_i(t+1) - \widetilde{I}_i(t+1) + \widetilde{V}_i(t+1)}{1 + k_u(t+1)}$$

(16.9) $$\widetilde{Y}_{di} = \left[\left(\widetilde{EBIT} - rD_c\right)(1-\tau_c) - rD_{pi}\right](1-\tau_{pi})$$

(16.10) $$\widetilde{Y}_{gi} = (\widetilde{EBIT} - rD_c)(1-\tau_c)(1-\tau_{gi}) - rD_{pi}(1-\tau_{pi})$$

(16.26) $$S_1 - E(S_1) = \varepsilon_1 \left[1 + \frac{\gamma}{1+k}\right] = [EBIT_1 - E_0(EBIT_1)]\left[1 + \frac{\gamma}{1+k}\right]$$

(16.27) $$\Delta Div_{it} = a_i + c_i(Div^*_{it} - Div_{i,t-1}) + U_{it}$$

(16.33) $$\Delta Div_t = \beta_1 Div_{t-1} + \beta_2 NI_t + \beta_3 NI_{t-1} + Z_t$$

(16.35) $$P_{it} = a + bDiv_{it} + cRE_{it} + \varepsilon_{it}$$

(16.38) $$\widetilde{R}_j = \gamma_0 + \left[\widetilde{R}_m - \gamma_0\right]\beta_j + \gamma_1\left[DY_j - DY_m\right]/DY_m + \varepsilon_j$$

(16.43) $$\frac{\Delta W}{N_0 P_0} = (1-F_P)\left(\frac{P_E - P_0}{P_0}\right) + F_P \frac{P_T - P_0}{P_0}$$

Hull: Options Futures and Other Derivatives

Chapter 12: Binomial Trees

(12.1) $$\Delta = \frac{f_u - f_d}{S_0 u - S_0 d}$$

(12.5/6) $$f = e^{-r\Delta t}\left[p f_u + (1-p) f_d\right] \qquad \text{where} \quad p = \frac{e^{r\Delta t} - d}{u - d}$$

(12.13/14) $$u = e^{\sigma\sqrt{\Delta t}} \qquad d = e^{-\sigma\sqrt{\Delta t}}$$

Chapter 13: Wiener Processes and Itôs Lemma

(13.17) $$G = \ln S \qquad dG = \left(\mu - \sigma^2/2\right) dt + \sigma\, dz$$

Chapter 14: The Black-Scholes-Merton model

(14.9) $$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right) dt + \frac{\partial f}{\partial S}\sigma S dz$$

(14.16) $$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

Chapter 18: Greek Letters

(18.) $$\Delta(\text{call}) = N(d_1) \qquad \Delta(\text{put}) = N(d_1) - 1$$

(18.2) $$\Theta(\text{call}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rKe^{-rT}N(d_2)$$

(18.2) $$\Theta(\text{put}) = -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rKe^{-rT}N(-d_2)$$

(18.) $$\Gamma(\text{call}) = \Gamma(\text{put}) = \frac{N'(d_1)}{S_0\,\sigma\sqrt{T}}$$

(18.4) $$\Theta + rS\Delta + \frac{1}{2}\sigma^2 S^2 \Gamma = r\Pi$$

(18.) $$\mathcal{V}(\text{call}) = \mathcal{V}(\text{put}) = S_0\sqrt{T}N'(d_1)$$

(18.) $$\text{rho(call)} = KTe^{-rT}N(d_2)$$

(18.) $$\text{rho(put)} = -KTe^{-rT}N(-d_2)$$

Chapter 20: Basic Numerical Procedures

(20.8) $$\Delta = \frac{f_{1,1} - f_{1,0}}{S_0 u - S_0 d}$$

(20.9) $\Gamma = \dfrac{(f_{2,2} - f_{2,1})/(S_0u^2 - S_0) - (f_{2,1} - f_{2,0})/(S_0 - S_0d^2)}{h}$

(20.10) $\Theta = \dfrac{f_{2,1} - f_{0,0}}{2\Delta t}$

(20.12) $p = \dfrac{e^{[f(t)-g(t)]\Delta t} - d}{u - d}$

(20.27) $a_j f_{i,j-1} + b_j f_{i,j} + c_j f_{i,j+1} = f_{i+1,j}$

where $a_j = \frac{1}{2}(r-q)j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t,\ b_j = 1 + \sigma^2 j^2 \Delta t + r\Delta t$

and $c_j = -\frac{1}{2}(r-q)j\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t$

(20.34) $f_{i,j} = a_j^* f_{i+1,j-1} + b_j^* f_{i+1,j} + c_j^* f_{i+1,j+1}$

where $a_j^* = \dfrac{1}{1+r\Delta t}\left(-\frac{1}{2}(r-q)j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right), \quad b_j^* = \dfrac{1}{1+r\Delta t}\left(1 - \sigma^2 j^2 \Delta t\right)$

and $c_j^* = \dfrac{1}{1+r\Delta t}\left(\frac{1}{2}(r-q)j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$

(20.35) $\alpha_j f_{i,j-1} + \beta_j f_{i,j} + \gamma_j f_{i,j+1} = f_{i+1,j}$

where $\alpha_j = \dfrac{\Delta t}{2\Delta Z}\left(r - q - \dfrac{\sigma^2}{2}\right) - \dfrac{\Delta t}{2\Delta Z^2}\sigma^2 \qquad \beta_j = 1 + \dfrac{\Delta t}{\Delta Z^2}\sigma^2 + r\Delta t$

and $\gamma_j = \dfrac{-\Delta t}{2\Delta Z}\left(r - q - \dfrac{\sigma^2}{2}\right) - \dfrac{\Delta t}{2\Delta Z^2}\sigma^2$

(20.36) $\alpha_j^* f_{i+1,j-1} + \beta_j^* f_{i+1,j} + \gamma_j^* f_{i+1,j+1} = f_{i,j}$

where $\alpha_j^* = \dfrac{1}{1+r\Delta t}\left[-\dfrac{\Delta t}{2\Delta Z}\left(r - q - \dfrac{\sigma^2}{2}\right) + \dfrac{\Delta t}{2\Delta Z^2}\sigma^2\right]$

and $\beta_j^* = \dfrac{1}{1+r\Delta t}\left(1 - \dfrac{\Delta t}{\Delta Z^2}\sigma^2\right)$

and $\gamma_j^* = \dfrac{1}{1+r\Delta t}\left[\dfrac{\Delta t}{2\Delta Z}\left(r - q - \dfrac{\sigma^2}{2}\right) + \dfrac{\Delta t}{2\Delta Z^2}\sigma^2\right]$

Chapter 22: Estimating Volatilities and Correlations

(22.7) $\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$

(22.8) $\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$

(22.12) $\displaystyle\sum_{i=1}^{m}\left[-\ln(v_i) - \frac{u_i^2}{v_i}\right]$

(22.13) $\mathrm{E}\left[\sigma_{n+t}^2\right] = V_L + (\alpha+\beta)^t\left(\sigma_n^2 - V_L\right)$

Chapter 25: Exotic Options

Call on a call

$$S_0e^{-qT_2}M\left(a_1,b_1;\sqrt{T_1/T_2}\right)-K_2e^{-rT_2}M\left(a_2,b_2;\sqrt{T_1/T_2}\right)-e^{-rT_1}K_1N(a_2)$$

$$\text{where}\quad a_1=\frac{\ln(S_0/S^*)+(r-q+\sigma^2/2)T_1}{\sigma\sqrt{T_1}}\quad a_2=a_1-\sigma\sqrt{T_1}$$

$$b_1=\frac{\ln(S_0/K_2)+(r-q+\sigma^2/2)T_2}{\sigma\sqrt{T_2}}\quad b_2=b_1-\sigma\sqrt{T_2}$$

Put on a call

$$K_2e^{-rT_2}M\left(-a_2,b_2;-\sqrt{T_1/T_2}\right)-S_0e^{-qT_2}M\left(-a_1,b_1;-\sqrt{T_1/T_2}\right)+e^{-rT_1}K_1N(-a_2)$$

Call on a put

$$K_2e^{-rT_2}M\left(-a_2,-b_2;\sqrt{T_1/T_2}\right)-S_0e^{-qT_2}M\left(-a_1,-b_1;\sqrt{T_1/T_2}\right)-e^{-rT_1}K_1N(-a_2)$$

Put on a put

$$S_0e^{-qT_2}M\left(a_1,-b_1;-\sqrt{T_1/T_2}\right)-K_2e^{-rT_2}M\left(a_2,-b_2;-\sqrt{T_1/T_2}\right)+e^{-rT_1}K_1N(a_2)$$

Chooser

$$\max(c,p)=c+e^{-q(T_2-T_1)}\max\left(0,Ke^{-(r-q)(T_2-T_1)}-S_1\right)$$

Barrier Options

$$\lambda=\frac{r-q+\sigma^2/2}{\sigma^2}\qquad y=\frac{\ln\left[H^2/(S_0K)\right]}{\sigma\sqrt{T}}+\lambda\sigma\sqrt{T}$$

$$x_1=\frac{\ln(S_0/H)}{\sigma\sqrt{T}}+\lambda\sigma\sqrt{T}\quad y_1=\frac{\ln(H/S_0)}{\sigma\sqrt{T}}+\lambda\sigma\sqrt{T}$$

If $H\le K$

$$c_{di}=S_0e^{-qT}(H/S_0)^{2\lambda}N(y)-Ke^{-rT}(H/S_0)^{2\lambda-2}N\left(y-\sigma\sqrt{T}\right)$$

If $H\ge K$

$$\begin{aligned}c_{do}&=S_0N(x_1)e^{-qT}-Ke^{-rT}N\left(x_1-\sigma\sqrt{T}\right)\\&\quad-S_0e^{-qT}(H/S_0)^{2\lambda}N(y_1)+Ke^{-rT}(H/S_0)^{2\lambda-2}N\left(y_1-\sigma\sqrt{T}\right)\end{aligned}$$

If $H>K$

$$\begin{aligned}c_{ui}=\;&S_0N(x_1)e^{-qT}-Ke^{-rT}N\left(x_1-\sigma\sqrt{T}\right)\\&-S_0e^{-qT}(H/S_0)^{2\lambda}\left[N(-y)-N(-y_1)\right]\\&+Ke^{-rT}(H/S_0)^{2\lambda-2}\left[N\left(-y+\sigma\sqrt{T}\right)-N\left(-y_1+\sigma\sqrt{T}\right)\right]\end{aligned}$$

If $H \geq K$

$$p_{ui} = -S_0 e^{-qT}(H/S_0)^{2\lambda} N(-y) + K e^{-rT}(H/S_0)^{2\lambda-2} N\left(-y + \sigma\sqrt{T}\right)$$

If $H \leq K$

$$\begin{aligned} p_{uo} = \ & -S_0 N(-x_1) e^{-qT} + K e^{-rT} N\left(-x_1 + \sigma\sqrt{T}\right) \\ & + S_0 e^{-qT}(H/S_0)^{2\lambda} N(-y_1) - K e^{-rT}(H/S_0)^{2\lambda-2} N\left(-y_1 + \sigma\sqrt{T}\right) \end{aligned}$$

If $H \leq K$

$$\begin{aligned} p_{di} = \ & -S_0 N(-x_1) e^{-qT} + K e^{-rT} N\left(-x_1 + \sigma\sqrt{T}\right) + S_0 e^{-qT}(H/S_0)^{2\lambda}\left[N(y) - N(y_1)\right] \\ & - K e^{-rT}(H/S_0)^{2\lambda-2}\left[N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right)\right] \end{aligned}$$

Lookback Options

$$c_{fl} = S_0 e^{-qT} N(a_1) - S_0 e^{-qT} \frac{\sigma^2}{2(r-q)} N(-a_1) - S_{\min} e^{-rT}\left(N(a_2) - \frac{\sigma^2}{2(r-q)} e^{Y_1} N(-a_3)\right)$$

where $a_1 = \frac{\ln(S_0/S_{\min}) + (r-q+\sigma^2/2)T}{\sigma\sqrt{T}}$ $\qquad a_2 = a_1 - \sigma\sqrt{T}$

$a_3 = \frac{\ln(S_0/S_{\min}) + (-r+q+\sigma^2/2)T}{\sigma\sqrt{T}}$

$Y_1 = \frac{2(r-q-\sigma^2/2)\ln(S_0/S_{\min})}{\sigma^2}$

$$p_{fl} = S_{\max} e^{-rT}\left(N(b_1) - \frac{\sigma^2}{2(r-q)} e^{Y_2} N(-b_3)\right) + S_0 e^{-qT} \frac{\sigma^2}{2(r-q)} N(-b_2) - S_0 e^{-qT} N(b_2)$$

where $b_1 = \frac{\ln(S_{\max}/S_0) + (-r+q+\sigma^2/2)T}{\sigma\sqrt{T}}$

$b_2 = b_1 - \sigma\sqrt{T}$

$b_3 = \frac{\ln(S_{\max}/S_0) + (r-q-\sigma^2/2)T}{\sigma\sqrt{T}}$

$Y_2 = \frac{2(r-q-\sigma^2/2)\ln(S_{\max}/S_0)}{\sigma^2}$

Exchange Options

(25.5) $\qquad V_0 e^{-q_V T} N(d_1) - U_0 e^{-q_U T} N(d_2)$

where $\qquad d_1 = \dfrac{\ln(V_0/U_0) + (q_U - q_V + \hat{\sigma}^2/2)T}{\hat{\sigma}\sqrt{T}} \qquad d_2 = d_1 - \hat{\sigma}\sqrt{T}$

and $\qquad \hat{\sigma} = \sqrt{\sigma_U^2 + \sigma_V^2 - 2\rho\sigma_U\sigma_V}$

Realized volatility: $\qquad \bar{\sigma} = \sqrt{\dfrac{252}{n-2}\displaystyle\sum_{i=1}^{n-1}\left[\ln\left(\frac{S_{i+1}}{S_i}\right)\right]^2}$

(25.6) $$\hat{E}(\bar{V}) = \frac{2}{T}\ln\frac{F_0}{S^*} - \frac{2}{T}\left[\frac{F_0}{S^*} - 1\right] + \frac{2}{T}\left[\int_{K=0}^{S^*}\frac{1}{K^2}e^{rT}p(K)dK + \int_{K=S^*}^{\infty}\frac{1}{K^2}e^{rT}c(K)dK\right]$$

(25.7) $L_{var}[\hat{E}(\bar{V}) - V_K]e^{-rT}$

(25.8) $$\int_{K=0}^{S^*} \frac{1}{K^2} e^{rT} p(K) dK + \int_{K=S^*}^{\infty} \frac{1}{K^2} e^{rT} c(K) dK = \sum_{i=1}^{n} \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i)$$

(25.9) $$\hat{E}(\bar{\sigma}) = \sqrt{\hat{E}(\bar{V})} \left\{ 1 - \frac{1}{8} \left[\frac{var(\bar{V})}{\hat{E}(\bar{V})^2} \right] \right\}$$

(25.10) $$\hat{E}(\bar{V})T = -\left(\frac{F_0}{S^*} - 1 \right)^2 + 2 \sum_{i=1}^{n} \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i)$$

Chapter 26: More on Models and Numerical Procedures

(26.2, 3) $$\frac{dS}{S} = (r-q)dt + \sqrt{V} dz_S \quad dV = a(V_L - V)dt + \xi V^{\alpha} dz_V$$

Chapter 27: Martingales and Measures

(27.4) $\Pi = (\sigma_2 f_2) f_1 - (\sigma_1 f_1) f_2$

(27.5) $\Delta\Pi = (\mu_1 \sigma_2 f_1 f_2 - \mu_2 \sigma_1 f_1 f_2) \Delta t$

(27.7) $$\frac{df}{f} = \mu dt + \sigma dz$$

(27.8) $$\frac{\mu - r}{\sigma} = \lambda$$

(27.13) $$\mu - r = \sum_{i=1}^{n} \lambda_i \sigma_i$$

(27.14) $$d\left(\frac{f}{g}\right) = (\sigma_f - \sigma_g) \frac{f}{g} dz$$

(27.20) $f_0 = P(0,T) E_T(f_T)$

(27.23) $$s(t) = \frac{P(t,T_0) - P(t,T_N)}{A(t)}$$

(27.25) $$f_0 = A(0) E_A \left[\frac{f_T}{A(T)} \right]$$

(27.26) $c = P(0,T) E_T [\max(S_T - K, 0)]$

(27.31) $$f_0 = U_0 E_U \left[\max\left(\frac{V_T}{U_T} - 1, 0 \right) \right]$$

(27.32) $f_0 = V_0 N(d_1) - U_0 N(d_2)$

(27.35) $\alpha_v = \rho \sigma_v \sigma_w$

Chapter 28: Interest Rate Derivatives: The Standard Market Models

(28.1) $c = P(0,T)\left[F_B N(d_1) - K N(d_2)\right]$

(28.2) $p = P(0,T)\left[K N(-d_2) - F_B N(-d_1)\right]$

where $d_1 = \dfrac{\ln(F_B/K) + \sigma_B^2 T/2}{\sigma_B\sqrt{T}}$ $\quad d_2 = d_1 - \sigma_B\sqrt{T}$

(28.3) $F_B = \dfrac{B_0 - I}{P(0,T)}$

(28.4) $\sigma_B = D y_0 \sigma_y$

(28.7, Caplet) $L\delta_k P(0,t_{k+1})\left[F_k N(d_1) - R_K N(d_2)\right]$

where $d_1 = \dfrac{\ln(F_k/R_K) + \sigma_k^2 t_k/2}{\sigma_k\sqrt{t_k}}$ $\quad d_2 = d_1 - \sigma_k\sqrt{t_k}$

(28.8, Floorlet) $L\delta_k P(0,t_{k+1})\left[R_K N(-d_2) - F_k N(-d_1)\right]$

(28.10) $LA\left[s_0 N(d_1) - s_k N(d_2)\right]$ where $A = \dfrac{1}{m}\sum_{i=1}^{mn} P(0,T_i)$

Chapter 29: Convexity, Timing, and Quanto Adjustments

(29.1) $E_T(y_T) = y_0 - \dfrac{1}{2} y_0^2 \sigma_y^2 T \dfrac{G''(y_0)}{G'(y_0)}$

(29.2) $E_T(R_T) = R_0 + \dfrac{R_0^2 \sigma_R^2 \tau T}{1 + R_0\tau}$

(29.3) $\alpha_V = \rho_{VW}\sigma_V\sigma_W$

(29.4) $E_{T^*}(V_T) = E_T(V_T)\exp\left[-\dfrac{\rho_{VR}\sigma_V\sigma_R R_0(T^* - T)}{1 + R_0/m} T\right]$

Chapter 30: Interest Rate Derivatives: Models of the Short Rate

(30.20)
$$\begin{aligned} c &= LP(0,s)N(h) - KP(0,T)N(h - \sigma_p) \\ p &= KP(0,T)N(-h + \sigma_p) - LP(0,s)N(-h) \\ h &= \frac{1}{\sigma_p}\ln\frac{LP(0,s)}{P(0,T)K} + \frac{\sigma_p}{2} \\ \sigma_p &= \frac{\sigma}{a}\left[1 - e^{-a(s-T)}\right]\sqrt{\frac{1 - e^{-2aT}}{2a}} \end{aligned}$$

(30.24) $P_{m+1} = \sum_{j=-n_m}^{n_m} Q_{m,j}\exp\left[-g\left(\alpha_m + j\Delta x\right)\Delta t\right]$

Chapter 32: Swaps Revisited

$$(32.1) \qquad F_i + \frac{F_i^2 \sigma_i^2 \tau_i t_i}{1 + F_i \tau_i}$$

$$(31.2) \qquad y_i - \frac{1}{2} y_i^2 \sigma_{y,i}^2 t_i \frac{G_i''(y_i)}{G_i'(y_i)} - \frac{y_i \tau_i F_i \rho_i \sigma_{y,i} \sigma_{F,i} t_i}{1 + F_i \tau_i}$$

$$(32.3) \qquad V_i + V_i \rho_i \sigma_{W,i} \sigma_{V,i} t_i$$

Hardy: Investment Guarantees

Chapter 2

AR(1) (2.7):

$$(2.7) \qquad \begin{aligned} Y_t &= \mu + a(Y_{t-1} - \mu) + \sigma \varepsilon_t \\ & \varepsilon_t \text{ independent and identically distributed (iid), } \varepsilon_t \sim N(0,1) \end{aligned}$$

ARCH(1) (2.8-9):

$$(2.8) \qquad Y_t = \mu + \sigma_t \varepsilon_t$$

$$(2.9) \qquad \sigma_t^2 = a_0 + a_1 (Y_{t-1} - \mu)^2$$

ARCH(1) with AR(1) stock price (2.10-11):

$$(2.10) \qquad \begin{aligned} Y_t &= \mu + a(Y_{t-1} - \mu) + \sigma_t \varepsilon_t \\ & \varepsilon_t \quad \text{iid} \sim N(0,1) \end{aligned}$$

$$(2.11) \qquad \sigma_t^2 = a_0 + \alpha (Y_{t-1} - \mu)^2$$

GARCH(1,1) (2.12-13):

$$(2.12) \qquad Y_t = \mu + \sigma_t \varepsilon_t$$

$$(2.13) \qquad \sigma_t^2 = a_0 + \alpha_1 (Y_{t-1} - \mu)^2 + \beta \sigma_{t-1}^2$$

GARCH(1,1) with AR(1) stock price (2.14-15):

$$(2.14) \qquad Y_t = \mu + a(Y_{t-1} - \mu) + \sigma_t \varepsilon_t \quad \varepsilon_t \text{ iid} \sim N(0,1)$$

$$(2.15) \qquad \sigma_t^2 = \alpha_0 + \alpha_1 (Y_{t-1} - \mu)^2 + \beta \sigma_{t-1}^2$$

Chapter 3

$$(3.1) \qquad L(\theta) = f(X_1, X_2, \ldots, X_n; \theta)$$

$$(3.2) \qquad L(\theta) = \prod_{t=1}^{n} f(x_t; \theta) \qquad\qquad l(\theta) = \sum_{t=1}^{n} \log f(x_t; \theta)$$

$$(3.3) \qquad L(\theta) = f(x_1;\theta)f(x_2;\theta|x_1)f(x_3;\theta|x_1,x_2)\cdots f(x_n;\theta|x_1,\ldots,x_{n-1})$$

$$(3.4) \qquad l(\theta) = \sum_{t=1}^{n} \log f(x_t;\theta|x_1,\ldots,x_{t-1})$$

$$(3.5) \qquad E[Y_t] = \mu \quad \text{for all } t$$

$$(3.6) \qquad E[(Y_t-\mu)(Y_{t-j}-\mu)] = \gamma_j \quad \text{for all } t \text{ and } j$$

$$(3.7) \qquad b(\theta) = E[\hat{\theta}-\theta]$$

Lognormal model (3.13-14,22-23):

$$(3.13) \qquad \hat{\mu} = \bar{y}$$

$$(3.14) \qquad \hat{\sigma} = \sqrt{\frac{\sum_{t=1}^{n}(y_t-\hat{\mu})^2}{n}}$$

$$(3.22) \qquad \Sigma = \begin{pmatrix} \sigma^2/n & 0 \\ 0 & \sigma^2/2n \end{pmatrix}$$

$$(3.23) \qquad \Sigma \approx \begin{pmatrix} \hat{\sigma}^2/n & 0 \\ 0 & \hat{\sigma}^2/2n \end{pmatrix}$$

AR(1) model (3.24-27):

$$(3.24) \qquad Y_t|Y_{t-1} \sim N(\mu(1-a)+aY_{t-1},\sigma^2) \quad t=2,3,\ldots,n$$

$$\begin{aligned}
(3.25,26) \qquad l(\mu,\sigma,a) &= \log\left(\sqrt{\frac{1-a^2}{2\pi\sigma^2}}\exp\left\{-\frac{1}{2}\left(\frac{(Y_1-\mu)^2(1-a^2)}{\sigma^2}\right)\right\}\right) \\
&\quad + \sum_{t=2}^{n}\log\left(\sqrt{\frac{1}{2\pi\sigma^2}}\exp\left\{-\frac{1}{2}\left(\frac{(Y_t-(1-a)\mu-aY_{t-1})^2}{\sigma^2}\right)\right\}\right) \\
&= -\frac{n}{2}\log(2\pi)+\frac{1}{2}\log(1-a^2)-n\log\sigma \\
&\quad -\frac{1}{2}\left\{\left(\frac{(Y_1-\mu)^2(1-a^2)}{\sigma^2}\right)+\sum_{t=2}^{n}\left(\frac{(Y_t-(1-a)\mu-aY_{t-1})^2}{\sigma^2}\right)\right\}
\end{aligned}$$

$$(3.27) \qquad V[\hat{\mu}] \approx \frac{\sigma^2}{n(1-a^2)} \quad V[\hat{\sigma}] \approx \frac{\sigma^2}{2n} \quad V[\hat{a}] \approx \frac{1-a^2}{n}$$

ARCH(1) (3.28-30):

$$(3.28) \qquad Y_t = \mu + \sigma_t\varepsilon_t$$

$$(3.29) \qquad \sigma_t^2 = a_0 + a_1(Y_{t-1}-\mu)^2$$

$$(3.30) \qquad Y_t|Y_{t-1} \sim N\left(\mu, a_0+a_1(Y_{t-1}-\mu)^2\right) \quad t=2,3,\ldots,n$$

GARCH(1,1) (3.31-33):

(3.31) $Y_t = \mu + \sigma_t \varepsilon_t$

(3.32) $\sigma_t^2 = \alpha_0 + \alpha_1 (Y_{t-1} - \mu)^2 + \beta \sigma_{t-1}^2$

(3.33) $Y_t | Y_{t-1} \sim N(\mu, \sigma_t^2) \quad t = 2, 3, \ldots, n$

(3.35) $\tilde{\mu} = \bar{y} \quad \tilde{\sigma} = s_y$

Chapter 8

(8.) $F_T = F_0 \dfrac{S_T}{S_0} (1-m)^T$

(8.3) $P_0 = G e^{-rT} \Phi(-d_2) - S_0 (1-m)^T \Phi(-d_1)$

$$\text{where } \; d_1 = \frac{\log\left(S_0\,(1-m)^T/G\right) + (r + \sigma^2/2)T}{(\sigma\sqrt{T})} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

(8.8) $H(0) = \int_0^n \left(G e^{-rt} \Phi(-d_2)\right) {}_t p_x^{\tau}\, \mu_{x,t}^{(d)}\, dt + \int_0^n \left(-S_0 (1-m)^t \Phi(-d_1)\right) {}_t p_x^{\tau}\, \mu_{x,t}^{(d)}\, dt$

(8.9)
$$\begin{aligned} H(0, t_3) \;=\; & \left(P_S(t_1) + S_0(1-m)^{t_1}\right) \times \\ & \left\{1 + P(t_2 - t_1)\left(1 + P(t_3 - t_2)\right) + (1-m)^{t_2 - t_1} P(t_3 - t_2)\right\} \\ & \quad -S_0(1-m)^{t_1} \end{aligned}$$

$$\text{where } \; P_S(t_1) = \mathrm{BSP}(S_0(1-m)^{t_1}, G, t_1)$$

(8.15) $\alpha = \dfrac{B}{S_0\, \ddot{a}^{\tau}_{x:\overline{n}|i'}}$

(8.19) $\mathrm{HE}_t = H(t) + {}_{t-1|}q_x^d \left((G - F_t)^+\right) - H(t^-)$

(8.22) $\mathrm{TC}_t = \tau\, S_t\, |\Psi_t - \Psi_{t-1}|$

FET-108-07 and FET-165-08: Doherty; Integrated Risk Management
Chapter 13

($p.$ 465)
$$\begin{aligned} V(E) &= V(F) - V(D) \\ &= V(F) - D_{DF} + P(V(F), D) \\ &= C(V(F), D) \end{aligned}$$

($p.$ 481) $V^*(F) = S + D\left(1 + \dfrac{m}{n}\right)$

($p.$ 494)
$$\begin{aligned} V_R'(E) &= -C + V_R(F) - D + P\left\{V_r(F), D\right\} \\ &= -C + V_R - D + P_R \\ V_N'(E) &= V_N(F) - D + P\left\{V_N(F), D\right\} \\ &= V_N - D + P_N \end{aligned}$$

Chapter 16

$$(16.1) \qquad T + (E + P)(1 + r_i) - L$$

$$(16.2) \qquad E(T) = (E - P - R(I;S))\,(1 + E(R_i)) - E(L(a)) + hC(I;S) - a$$

$$(16.3) \qquad \frac{\partial E(T)}{\partial a} = \frac{-\partial E(L(a))}{\partial a} - 1 + h\frac{\partial C}{\partial I}\frac{\partial I}{\partial L}\frac{\partial L}{\partial a} = 0$$

Manistre and Hancock; Variance of the CTE Estimator

$$(p.\,130) \qquad C\hat{T}E(\alpha) = \frac{1}{k}\sum_{j=1}^{k} x_{(j)}$$

$$(p.\,131) \qquad VAR(C\hat{T}E) = E[VAR(C\hat{T}E|X_{(k)})] + VAR[E(C\hat{T}E|X_{(k)})]$$

$$(p.\,131) \qquad VAR(C\hat{T}E) \approx \frac{VAR(x_{(1)},\ldots,x_{(k)}) + \alpha\cdot(C\hat{T}E - x_{(k)})^2}{k}$$

$$(p.\,132) \qquad C\hat{T}E(\alpha) = \frac{1}{k}\sum_{i=1}^{k} x_{(i)} \qquad V\hat{a}R(\alpha) = x_{(k)}$$

$$(p.\,133) \qquad IF_{VaR}(x) = \begin{cases} \frac{-(1-\alpha)}{f(VaR)} & x < VaR \\ 0 & x = VaR \\ \frac{\alpha}{f(VaR)} & x > VaR \end{cases}$$

$$(p.\,133) \qquad IF_{CTE}(x) = \begin{cases} VaR - CTE & x < VaR \\ VaR - CTE + \frac{x - VaR}{1-\alpha} & x > VaR \end{cases}$$

$$(p.\,133) \qquad VAR(C\hat{T}E_n) \approx \frac{E[IF_{CTE})^2]}{n} = \frac{VAR(X|X > VaR) + \alpha\cdot(CTE - VaR)^2}{n\cdot(1-\alpha)}$$

$$(p.\,133) \qquad VAR(V\hat{a}R_n) \approx \frac{E[IF_{VaR})^2]}{n} = \frac{\alpha\cdot(1-\alpha)}{n\cdot[f(VaR)]^2}$$

$$(p.\,133) \qquad Cov(C\hat{T}E_n, V\hat{a}R_n) \approx \frac{E[IF_{CTE}\cdot IF_{VaR}]}{n} = \frac{\alpha\cdot(CTE - VaR)}{n\cdot f(VaR)}$$

$$(p.\,134) \qquad FSE(CTE) = \sqrt{\frac{VAR(X_{(1)},\ldots,X_{(k)}) + \alpha\cdot(C\hat{T}E - X_{(k)})^2}{n\cdot(1-\alpha)}}$$

$$(p.\,134) \qquad FSE(VaR) = \frac{1}{\hat{f}(VaR)} \cdot \sqrt{\frac{\alpha \cdot (1-\alpha)}{n}}$$

$$(p.\,134) \qquad C\hat{o}v(CTE, VaR) = \frac{\alpha \cdot (C\hat{T}E - X_{(k)})}{n \cdot \hat{f}(V\hat{a}R)}$$

$$(p.\,146) \qquad VAR(V\hat{a}R_n) \approx \frac{E_G[(IF_{VaR})^2]}{n} = \frac{VAR_G[WH]}{n \cdot [f(VaR)]^2}$$

$$(p.\,146) \qquad VAR(C\hat{T}E_n) \approx \frac{E_G[(IF_{CTE})^2]}{n} = \frac{VAR_G[W(X - VaR)H]}{n \cdot (1-\alpha)^2}$$

$$(p.\,146) \qquad Cov(C\hat{T}E_n, V\hat{a}R_n) \approx \frac{E_G[IF_{CTE} \cdot IF_{VaR}]}{n} = \frac{Cov_G[WH, WH(X - VaR)]}{n \cdot f(VaR) \cdot (1-\alpha)}$$

$$(p.\,146) \qquad VAR(V\hat{a}R_n) \approx (1-\alpha) \cdot \frac{E_F[W|X \geq VaR] - 1 + \alpha}{n \cdot [f(VaR)]^2}$$

$$\begin{aligned}(p.\,146) \qquad VAR(C\hat{T}E_n) \;\; &\approx \;\; \frac{VAR_F[\sqrt{W}H(X - VaR)]}{n \cdot (1-\alpha)^2} \\ &= \;\; \frac{E_F[W|X \geq VaR] \cdot VAR_F[X| \geq VaR]}{n \cdot (1-\alpha)} \\ &\quad + \frac{(CTE - VaR)^2 \cdot (E_F[W|X \geq VaR] - 1 + \alpha)}{n \cdot (1-\alpha)} \\ &\quad + \frac{Cov_F[W, (X - VaR)^2 | X \geq VaR]}{n \cdot (1-\alpha)}\end{aligned}$$

$$\begin{aligned}(p.\,146) \qquad Cov(C\hat{T}E, V\hat{a}R) \;\; &\approx \;\; \frac{Cov_F[W, X - VaR | X \geq VaR]}{n \cdot f(VaR)} \\ &\quad + \frac{1-\alpha}{n \cdot f(VaR)} \cdot \{E_F[W|X \geq VaR] - 1 + \alpha\}\end{aligned}$$

FET-106-07: Ho and Lee, The Oxford Guide to Financial Modeling
Chapter 5: Interest Rate Derivatives: Interest Rate Models

(5.10)
$$\begin{aligned} dr &= a_1 + b_1(l - r)dt + r\sigma_1 dZ \\ dl &= (a_2 + b_2 r + c_2 l)\, dt + l\sigma_2 dW \end{aligned}$$

(5.12)
$$\begin{aligned} &dV = M(t,r)dt + \Omega(t,r)dZ \\ &M(t,r) = V_t + \mu(t,r)V_r + \tfrac{1}{2}\sigma(t,r)^2 V_{rr} \\ &\Omega(t,r) = \sigma(t,r)V_r \end{aligned}$$

Chapter 6: Implied Volatility Surface: Calibrating the Models

(6.10)
$$\begin{aligned} &L(k, j+1) \\ &\quad = L(k,j)\exp\left[\left(\sum_{i=j+1}^{k} \tfrac{L(i,j)\Delta}{1+L(i,j)\Delta}\Lambda_{i-j-1}\Lambda_{k-j-1} - \tfrac{\Lambda_{k-j-1}^2}{2}\right)\Delta + \Delta_{k-j-1}\sqrt{\Delta}Z\right] \end{aligned}$$

(6.13)
$$P(T^*, i\,; T) = \frac{P(T^*+T)}{P(T^*)}\, 2\, \frac{\prod_{t=T}^{T+T^*-1} h(t)}{\prod_{t=1}^{T^*-1} h(t)}\, \delta^{Ti} \quad \text{where } h(t) = \frac{1}{1+\delta^t}$$

FET-114-07: Capital Allocation in Financial Firms

*p.*116
$$\begin{aligned} NPV &= (1-d)V\{S^+\} - (C - \mu) - (1+m)V\{S^-\} \\ &= \mu - (dV\{S^+\} + mV\{S^-\}) \\ V\{S^+\} &= \tfrac{\sigma(n(z)+zN(z))}{(1+r)} \\ V\{S^-\} &= \tfrac{\sigma(n(z)-zN(-z))}{(1+r)} \\ z &= C(1+r)/\sigma \end{aligned}$$

FET-151-08: Babbel and Merrill, Real and Illusory Value Creation by Insurance Companies

(2)
$$V(E) = F + V(A_T) - PV(L) + O$$

Smith: Investor & Management Expectations of the Return on Equity Measure vs. Some Basic Truths of Financial Accounting

$$E_t - EV_t = \sum_{x=1}^{t}\left[(ROE_x - IRR)E_{x-1}(1+IRR)^{t-x}\right]$$